

Low Energy Expansion in the Three Body System to All Orders and the Triton Channel[1]

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The disparity between the energy scales of typical QCD phenomena and the scale for nuclear binding makes nuclear systems an ideal playing ground for Effective Field Theory (EFT) methods. Recently, considerable effort has been put into the development of a phenomenologically successful EFT for nuclear physics. It soon became apparent that there is one feature which distinguishes the nuclear EFT from most of the many other applications of EFT methods: The low energy expansion is non-perturbative in the sense that an infinite number of diagrams contribute at each order. This follows from the very existence of shallow real and virtual bound states whose binding energies are so small that they lie within the range of validity of the EFT even though they cannot be described by perturbing the free theory. In fact, there are at least in the two and three-nucleon sector states so loosely bound that their scales do not seem to be connected even to the soft QCD scales m_π or f_π . Instead, they appear to require an additional “accidental” fine-tuning whose microscopic origin is not understood. This phenomenon offers the opportunity for a more radical use of EFT in the few nucleon case. In this approach, all particles but the nucleon themselves are considered high energy degrees of freedom and are consequently “integrated out”. The resulting EFT is considerably simpler than potential models or the “pionful” version of nuclear EFT, but its range of validity is reduced to typical momenta below the pion mass. This approach is well established in the two-body sector. On a diagram-by-diagram basis, the graphs contributing at leading order (LO) to nucleon-deuteron scattering are finite. However, it turns out that the ultraviolet behaviour of the equation describing the three-body system in the ${}^3\text{He}$ - ${}^3\text{H}$ channel is very different from the behaviour of its perturbative expansion.

We presented a simple scheme to classify at which order a given three-body force must be considered in an EFT approach to the few body system at very low energies. We compared the predictions of this scheme in the case of an NNLO calculation of the nd scattering phase shifts in the ${}^2\text{S}_{1/2}$ (triton) channel. We confirmed

the well-known result that to LO and NLO, only one momentum dependent three-body force is necessary for cutoff independence, whose strength can be determined by the three-body scattering length. We found that one and only one new parameter enters at NNLO, namely the Wigner- $SU(4)$ -symmetric three-body force with two derivatives. The one additional datum needed to fix it is the binding energy of the triton. Indeed, only the $SU(4)$ -symmetric three-body forces are systematically enhanced over their contributions found from a naive dimensional estimate, with a $2n$ -derivative three-body force entering at the $2n$ th order. The phase shifts thus calculated are in good agreement with a partial wave analysis and more sophisticated model computations, as shown in Fig. 1.

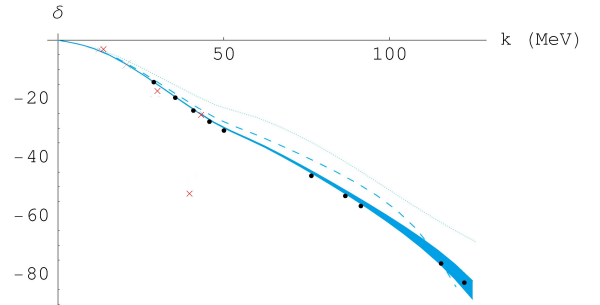


FIG. 1: The neutron-deuteron ${}^2\text{S}_{1/2}$ phase shift at LO (dotted line), NLO (dashed line) and NNLO (dark band). The band in the NNLO curve corresponds to the change caused by a variation of the cutoff from $\Lambda = 200$ MeV to $\Lambda = 600$ MeV. The dots are from phase shift analysis of the triton channel, and the crosses are the results obtained with the Argonne V18+Urbana IX two and three-body forces.

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